On Finding a Sparse Subgraph in Subclasses of Perfect Graphs ⋆

R. Watrigant, M. Bougeret, and R. Giroudeau

LIRMM-CNRS-UMR 5506 - 161, rue Ada 34090 Montpellier, France

Given a simple undirected graph $G = (V, E)$ and an integer $k \leq |V|$, the k-SPARSEST SUBGRAPH (k-SS) problem asks for a set of k vertices inducing[1] the minimum number of edges. As a generalization of the classical INDEPENDENT SET problem, k-SS is $\mathcal{NP}$-hard in general graphs as well as $W[1]$-hard and $O(n^{1-\epsilon})$-inapproximable. In addition, it is well known that INDEPENDENT SET is polynomial-time solvable in perfect graphs. Thus, it appears natural to investigate the complexity of k-SS in subclasses of perfect graphs. On the other side, k-SPARSEST SUBGRAPH is strongly connected to its complementary problem, namely the k-DENSEST SUBGRAPH problem (k-DS), which consists in finding a set of k vertices inducing the maximum number of edges. In [3], the authors show that k-DS is $\mathcal{NP}$-hard in bipartite, comparability and chordal graphs, and is polynomial-time solvable in trees, cographs, bounded treewidth and split graphs. The question of the complexity status of k-DS in interval graphs (and even in proper interval graphs) is stated by the authors as an open problem, and is still not answered yet. In addition, [2] shows that both k-SS and k-DS are polynomial time solvable in bounded cliquewidth graphs. Notice that several exact or approximation algorithm exists for k-DS in subclasses of perfect graphs: among others, constant approximation algorithms are known for chordal graphs [5], bipartite permutation graphs [1] and $PTAS$ are known for interval graphs [6] and for chordal graphs having a special clique tree [4].

In this talk we will sketch two dynamic programming algorithms for k-SS, leading to a $PTAS$ in proper interval graphs and an FPT algorithm in interval graphs (parameterized by the number of edges in the solution, which is a stronger parameterization than the classical one, *i.e.* by k). In both cases, we use a decomposition of the graph into a path of separators, and use restructuration arguments to retrieve efficiently good solutions. We will also sketch the $\mathcal{NP}$-hardness of k-SS in chordal graphs and discuss interesting open problems. All of our results are available in [7,8].

References

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[1] An edge $\{u, v\}$ is said to be induced (resp. covered) by a set S of vertices if $u \in S$ and $v \in S$ (resp. $u \in S$ or $v \in S$).